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Differential Calculus- (Partial differentiation)

Q.1 Show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ when

$$u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$$

Soln:→ We have, $u=\sin^{-1}\left(\frac{x}{y}\right)+\tan^{-1}\left(\frac{y}{x}\right)$ —— (I)

Differentiating (I) with respect to x, we get

$$\frac{\partial u}{\partial x}=\frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\cdot\frac{1}{y}+\frac{1}{1+\frac{y^2}{x^2}}\left(-\frac{y}{x^2}\right)$$

$$\Rightarrow \frac{\partial u}{\partial x}=\frac{1}{\sqrt{y^2-x^2}}-\frac{y}{x^2+y^2}$$

$$\therefore x\frac{\partial u}{\partial x}=\frac{x}{\sqrt{y^2-x^2}}-\frac{xy}{x^2+y^2} \quad \text{—— (II)}$$

Again, differentiating I with respect to y, we get

$$\frac{\partial u}{\partial y}=\frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\cdot\frac{-x}{y^2}+\frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{1}{x}$$

$$=\frac{-x}{y\sqrt{y^2-x^2}}+\frac{x}{x^2+y^2}$$

$$\therefore y\frac{\partial u}{\partial y}=\frac{-xy}{y\sqrt{y^2-x^2}}+\frac{xy}{x^2+y^2} \quad \text{—— (III)}$$

Adding II and III, we get

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\cancel{\frac{x}{\sqrt{y^2-x^2}}}-\cancel{\frac{xy}{x^2+y^2}}-\cancel{\frac{x}{\sqrt{y^2-x^2}}}+\cancel{\frac{xy}{x^2+y^2}}=0$$

Proved

Q.2. If $u = \log \dfrac{x^2+y^2}{x-y}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$

Soln We have, $u = \log\left(\dfrac{x^2+y^2}{x+y}\right) = \log(x^2+y^2) - \log(x+y)$ —I

Differentiating I with respect to x, we get

$$\frac{\partial u}{\partial x} = \frac{1}{x^2+y^2}\cdot 2x - \frac{1}{x+y}\cdot 1$$

$$\therefore x\frac{\partial u}{\partial x} = \frac{2x^2}{x^2+y^2} - \frac{x}{x+y} \quad \text{———— (II)}$$

Also, differentiating I with respect to y, we get

$$\frac{\partial u}{\partial y} = \frac{2y}{x^2+y^2} - \frac{1}{x+y}$$

$$\therefore y\frac{\partial u}{\partial y} = \frac{2y^2}{x^2+y^2} - \frac{y}{x+y} \quad \text{———— (I_II)}$$

Adding II and III, we get

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\left[\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2}\right] - \left[\frac{x}{x+y} + \frac{y}{x+y}\right]$$

$$= 2\,\cancel{\frac{x^2+y^2}{x^2+y^2}} - \cancel{\frac{x+y}{x+y}} = 2-1 = 1.$$

Proved.

Q.3. If $u = 3x^2yz + 5xy^2z + 4z^4$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 4u.$$

Soln: We have, $u = 3x^2yz + 5xy^2z + 4z^4$

$$\therefore \frac{\partial u}{\partial x} = 6xyz + 5y^2z, \quad \frac{\partial u}{\partial y} = 3x^2z + 10xyz$$

and $$\frac{\partial u}{\partial z} = 3x^2y + 5xy^2 + 16z^3$$

Now, $L.H.S = x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z} = 6x^2yz + 5xy^2z + 3x^2yz + 10xy^2z + 3x^2yz + 5xy^2z + 16z^4$

$$= 12x^2yz + 20xy^2z + 16z^4$$

$$= 4(3x^2yz + 5xy^2z + 4z^4) = 4u = R.H.S$$

proved